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U. S.
DAHLGREN, VIRGINIA

The Theory of Projectile Ricochet

by

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Computation and Ballistics Department

NPG REPORT NO. 1041
Task Assignment No. NPG K-11011-1
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ABSTRACT

Force-pen[illegible]ation curves for 3" AP M79 proj[illegible] [illegible] obliquity are derived with the aid of simplifying assumpt[illegible] as [illegible] stretching and folding of the petals of [illegible] plate. The [illegible] [illegible] are compared with experimental [illegible]. [illegible] analysis [illegible] [illegible] effe[illegible] of incomplete contact between [illegible] [illegible] [illegible] [illegible] analy[illegible] is guided by an experiment w[illegible] [illegible] [illegible]val indent[illegible] [illegible] foil.

A theory is developed to represent the [illegible] of a thin membrane at low obliquity. The theory is [illegible] [illegible] following [illegible]fications:

(a) It is assumed that the projec[illegible] [illegible] complete contact with the membrane until the [illegible] [illegible] [illegible]ches the membrane.

(b) It is assumed that a transv[illegible] [illegible] brings the velocity of the center of mass instantane[illegible] [illegible] coincidence with the axis of the projectile when the [illegible] first touches the membrane.

(c) It is assumed that the projec[illegible]e continues without transverse force or torque after the b[illegible]elet has first touched the membrane.

(d) It is assumed that the f[illegible] deflection of the velocity of the center of mass [illegible] [illegible]ate in the penetration to be of influence on the limit ve[illegible]ity for perforation.

(c) It is assumed [illegible] the [illegible]tion in the undulation is the same as the motion in [illegible] [illegible] undulation with the force concentrated at a point.

[redacted] [redacted]

The force and torque on a 3" AP M79 projectile at high obliquity are estimated with the aid of simplifying assumptions as to the plastic deformation at the sides of a crack.

A theory is developed to represent the limiting case of a thin membrane at high obliquity. The theory is based on the following simplifications:

(a) It is assumed that the force and torque on the nose of the projectile are constant until the axis of the projectile turns out of the membrane.

(b) It is assumed that the force and torque on the base of the projectile are constant, after the base reaches the membrane, until the axis of the projectile turns out of the membrane.

(c) It is assumed that the final force and torque on the projectile are given by the limiting functions for a projectile with its axis nearly perpendicular to the membrane.

(d) It is assumed that the motion in the undulation is the same as the motion in an elastic undulation with the force concentrated at a point until the axis of the projectile turns out of the membrane. It is assumed that the displacement of the undulation is stationary while the projectile see-saws in the membrane.

The effect of yaw and the effect of rate of yaw on the limit energy is estimated. The role of cavitation is described.

[redacted]

FOREWORD

The material in this report is basic to the construction of plate penetration charts or tables. It was originally authorized by BUORD ltr NP9/A9(Re3) of 9 January 1943, was later charged to Task Assignment NPG-41-Re3a-118-1, and is currently charged to the Foundational Research Program of the Naval Proving Ground.

The material in this report has been prepared since World War II in connection with a study of the mechanism of penetration of plate by projectiles. The report is one of a series of reports. Five of the reports were published at the end of the war, and it was originally planned that nine reports would be submitted altogether. The remaining four reports were held up pending a revaluation of the ballistic data, inasmuch as there was an opportunity to obtain a few additional tests of special interest at the end of the war. As a result of these tests, the number of reports has been increased to eleven. The six remaining reports are now to be published, but with a minimum expenditure of additional effort in order to bring forth the existing material. The analysis has probably been carried as far as it should be carried without the aid of a modern calculator such as the Aiken Dahlgren Electronic Calculator. The press of urgent work has thus far prevented allocation of any ADEC time to this work.

[redacted]

The titles of the full set of eleven reports are as follows:

(1) ANALYTICAL SUMMARY. PART I. THE PHYSICAL [redacted] STS UNDER TRIAXIAL STRESS. NPG Report No. 6-46

<u>Object</u>: To summarize the available data on the physical properties of Class B Armor and STS under triaxial stress.

(2) ANALYTICAL SUMMARY. PART II. ELASTIC AND PLASTIC UNDULATIONS IN ARMOR PLATE. NPG Report No. 7-46

<u>Object</u>. To analyse the propagation of undulations in armor plate; to summarize previous analytical work and to add new analytical work where required in order to complete the theory for ballistic applications.

(3) ANALYTICAL SUMMARY. PART III. PLASTIC FLOW IN ARMOR PLATE. NPG Report No. 864

<u>Object</u>: To analyse the plastic flow in armor plate adjacent to the point of impact by a projectile.

(4) ANALYTICAL SUMMARY. PART IV. THE THEORY OF ARMOR PENETRATION. NPG Report No. 9-46

<u>Object</u>: To summarize the theory of armor penetration in its present state of development, and to develop theoretical functions which can be used as a guide in the interpretation of ballistic data.

(5) ANALYTICAL SUMMARY. PART V. PLASTIC FLOW IN BARS AND SHELLS. NPG Report No. 954

<u>Object</u>: To analyse the plastic flow in cylindrical bars and shells during impact against an unyielding plate.

(6) ANALYTICAL SUMMARY. PART VI. THE THEORY OF PROJECTILE RICOCHET. NPG Report No. 1041

<u>Object</u>: To analyse the dynamics of projectiles during oblique impact, and to develop theoretical functions which can be used as a guide in the interpretation of ballistic data.

(7) BALLISTIC SUMMARY. PART I. THE DEPENDENCE OF LIMIT VELOCITY ON PLATE THICKNESS AND OBLIQUITY AT LOW OBLIQUITY. NPG Report No. 2-46

<u>Object</u>: To compare the results of ballistic test with the prediction of existing formulae, and with the results of theoretical analysis; to find the mathematical functions which best represent the fundamental relationship between limit velocity, plate thickness, and obliquity at low obliquity.

(8) BALLISTIC SUMMARY. PART II. THE SCALE EFFECT AND THE OGIVE EFFECT. NPG Report No. 4-46

<u>Object</u>: To determine the effect of scale on ballistic performance, and to correlate the projectile nose shape with the results of ballistic test.

(9) BALLISTIC SUMMARY. PART III. THE WINDSHIELD EFFECT, THE HOOD EFFECT, AND THE CAP EFFECT. NPG Report No. 1211

<u>Object</u>: To determine the effect of windshields and hoods or caps on ballistic performance.

(10) BALLISTIC SUMMARY. PART IV. THE DEPENDENCE OF LIMIT VELOCITY ON PLATE THICKNESS AND OBLIQUITY AT HIGH OBLIQUITY. NPG Report No. 1125

<u>Object</u>: To compare the results of ballistic test with the results of theoretical analysis; to find mathematical functions which best represent the fundamental relationship between limit velocity, plate thickness, and obliquity at high obliquity.

(11) BALLISTIC SUMMARY. PART V. THE CONSTRUCTION OF PLATE PENETRATION CHARTS OR TABLES. NPG Report No. 1120

<u>Object</u>: To summarize the results of analysis in the form of standard charts or tables.

The computations for this report were performed by:
V. L. Nichols and J. M. Foster
Mathematical Physics Branch
Computation and Ballistics Department

This report was reviewed by:
N. A. M. Riffolt, Director of Research

INTRODUCTION

A theoretical analysis of the motion of a projectile during ricochet in armor plate has a twofold purpose. It forms a rational basis for the extrapolation of ballistic data to obliquities which exceed the range of ballistic test. It leads to an estimate of the effect of yaw and the effect of rate of yaw on the limit energy for perforation.

The limit energy per unit weight of armor in the path of a monobloc projectile is nearly independent of obliquity at low obliquity. The plate applies a torque to the projectile which increases with increase in obliquity. The torque deflects the axis of the projectile and gives the projectile an angular velocity. In a limit impact at high obliquity the projectile is actually turned into a position with its axis perpendicular to the plate and with the base of the projectile through the plate. The impact area extends a long distance beyond the point of impact in an impact at high obliquity. The limit energy per unit weight of armor in the path of the projectile varies with obliquity at high obliquity.

The penetration during an oblique impact may be divided into three stages. During stage (i) the projectile is in contact with the plate around the complete perimeter of the impact hole and the force and torque are independent of the direction of motion of the projectile. During stage (ii) the projectile partially loses contact with the plate and there are additional transverse forces and torques on the projectile. During stage (iii) the projectile moves sidewise in the plate.

An accurate analysis of the motion of the projectile would require a step by step numerical integration of the three simultaneous equations of motion, with an analysis of the area of contact between plate and projectile at each step. Inasmuch as the complexity of this numerical integration exceeds the capacity of available personnel, the analysis has been expedited, in the present report, with the aid of somewhat drastic simplifications.

The semiquantitative theory in its present state is applied in the present report to the penetration of 3" AP M79 projectiles at low obliquity and at high obliquity in a thin membrane of STS with a tensile strength of 115000 $(lb)/(in)^2$.

THE EQUATIONS OF MOTION

The position of the center of mass of the projectile after the first instant of contact with the plate may be expressed in terms of Cartesian coordinates x, y, z, of which the coordinate z is the distance of the center of mass from the initial plane of the plate, and is negative before impact. The Cartesian axes are unit vectors i, j, k, among which the vector i is perpendicular to the plane of incidence, the vector j is parallel to the plane of incidence and is parallel to the plate, and the vector k is perpendicular to the plate. The orientation of the projectile may be expressed in terms of the angle χ between the axis of the projectile and the normal of the plate. The direction of motion of the projectile may be expressed in terms of the angle θ between the velocity of the center of mass and the normal of the plate. The coordinates of the center of mass and the orientation of the projectile are illustrated in Figure (1).

The motion of the projectile is governed by the equations of motion

$$m\dot{\mathbf{v}} = \mathbf{f} \qquad C\dot{\omega} = \mathbf{L} \tag{1}$$

in which m is the mass, C is the transverse moment of inertia, $\mathbf{v}$ is the velocity of the center of mass, ω is the angular velocity, $\mathbf{f}$ is the resultant force on the projectile, and L is the resultant torque.

Let the dimensionless variables $g, h, q, \alpha, \gamma, \eta, \Lambda$ be defined by the equations

$$g = \frac{y}{d} \qquad\qquad h = \frac{z}{d} \tag{2}$$

$$q = \left(\frac{eX'}{m}\right)^{\frac{1}{2}} t \tag{3}$$

$$\alpha = \frac{md^2}{C} \tag{4}$$

$$\gamma = \frac{\mathrm{i} \cdot \mathrm{f}}{deX'}, \qquad\qquad \eta = \frac{\mathrm{k} \cdot \mathrm{f}}{deX'}, \tag{5}$$

$$\Lambda = -\frac{\mathrm{L}}{d^2 eX'} \tag{6}$$

in which d is the diameter of the projectile, e is the thickness of the plate, and X' is the yield stress of the plate. Substitution of the dimensionless variables into the equations of motion leads to the equations

$$g'' = \gamma \qquad\qquad h'' = \eta \tag{7}$$

$$\chi'' = + \alpha\Lambda \tag{8}$$

in which g'', h'', χ'' are derivatives with respect to q.

The general integrals of the equations of motion are given by the equations

$$g' = g_0' + \int_0^q \gamma dq \qquad\qquad h' = h_0' + \int_0^q \eta dq \tag{9}$$

$$\chi' = \chi_0' + \alpha \int_0^q \Lambda dq \tag{10}$$

and by the equations

$$g = g_0 + g_0' q + q\int_0^q \gamma dq - \int_0^q q\gamma dq$$

$$h = h_0 + h_0' q + q\int_0^q \eta dq - \int_0^q q\eta dq \qquad (11)$$

$$\chi = \chi_0 + \chi_0' q + \alpha q\int_0^q \Lambda dq - \alpha\int_0^q q\Lambda dq$$

in which g_0, h_0, χ_0, g_0', h_0', χ_0' are the values of g, h, χ, g', h', χ' at $q = 0$. The values of g, h, χ, g', h', χ' at the end of each stage of the motion are the values of g_0, h_0, χ_0, g_0', h_0', χ_0' for the beginning of the next stage.

The penetration p of the projectile into the plate is given by the equation

$$p = s - \xi + (b + \lambda)\cos\chi \qquad (12)$$

in which ξ is the displacement of the center of the undulation in the plate, λ is the distance from the center of mass of the projectile to the edge of the bourrelet, and b is the distance from the edge of the bourrelet to the tip of the nose. The displacement ξ is given approximately by the equation*

$$\frac{\xi}{d} = -\beta_1 + \sqrt{\beta_1^2 + \beta_2 (h_0' - h')} \qquad (13)$$

in which β_1, β_2 are dimensionless parameters. The parameters β_1, β_2 are defined by the equations

$$\beta_1 = (1.11)\frac{e}{d} \qquad \beta_2 = \frac{1}{(1.09)}\left(\frac{mI'}{\rho d^{3} E}\right)^{\frac{1}{2}}\left(\frac{e}{d}\right)^{-\frac{1}{2}} \qquad (14)$$

in which E is the Young's modulus and ρ is the density of the plate.

THE MOTION IN A LIMIT IMPACT ON A THIN MEMBRANE

The Force at Normal Obliquity

The force on a round nosed projectile during the penetration of a thin membrane has been analysed in reference (1). The analysis was based on the following simplifications.

(1) It was assumed that a star crack is formed at first contact and spreads out to the full diameter of the projectile with no expenditure of energy.

(2) It was assumed that each petal of the star is pushed back to form a cylindrical surface, while the rest of the membrane remains plane.

(3) It was assumed that the energy which is required to push back each petal is the same as the energy which would be required to warp a plane sector with a circular base into a plane triangle with a straight base.

The membrane is actually dished before fracture by a round nosed projectile, and the petals are not bent perfectly into cylindrical form. These two deviations tend to cancel, however, and the theory is in good agreement with the ballistic data. The possibility of a simultaneous distortion of the whole petal is attributed to the preliminary formation of a dish and to the round shape of the projectile.

The total plastic energy w of the penetration is given by the equation

$$w = \frac{2\pi}{N^2} a^2 eX' \tag{15}$$

in which a is the radius of the projectile and N is the number of petals in the star crack. A round nosed projectile normally makes a star crack with three petals, for which the plastic energy is given by the equation

$$w = (.222)\pi a^2 eX' \tag{16}$$

The membrane is actually fractured on contact by a pointed projectile, but the cracks and petals are formed by a progressive distortion. During a small displacement of the pointed projectile along its axis, a narrow annular zone of the membrane is folded back into each petal. As the projectile advances into the impact hole, the petals are pushed back from the point of the projectile. The energy which is required for the formation of a petal is probably analogous to the energy which would be required for the progressive transformation of a plane sector into a plane triangle. As the annular zone is folded back, the arc of the plane sector would be stretched, and as the whole petal is pushed back, the arc of the plane sector would be straightened.

The analysis of force on a pointed projectile during the penetration of a thin membrane is based on the following simplifications.

(a) It is assumed that a star crack is formed at the first instant of contact and spreads progressively with no expenditure of energy.

(b) It is assumed that the petals grow progressively while the rest of the membrane remains plane.

(c) It is assumed that the energy which is required to fold back an annular zone of the membrane into each petal is the same as the energy which would be required to stretch the base of a plane triangle.

(d) It is assumed that the energy which is required to push back each petal is the same as the energy which would be required to warp the arc of a plane sector.

During the differential displacement dp of an ogival projectile into the membrane, an annular zone of radius r and width dr is folded back into a conical zone which is tangent to the ogive. The base of each petal is stretched by the amount

$$\left(1 - \frac{1}{\sqrt{1 + \left(\frac{dp}{dr}\right)^2}}\right) \varphi dr = (1 - k\cdot n)\varphi dr \qquad (17)$$

in which the polar angle φ is measured from the median line of the petal and n is a unit vector normal to the ogive. The increment in plastic energy is given by the expression

$$\frac{\pi^2 \bar{\tau}}{N}(1 - k\cdot n) r dr$$

in which $\bar{\tau}$ is the average shear stress on the base of the petal.

During the differential displacement $d\phi$, the petals are pushed back, and the base line of each petal is bent. The plastic work is approximately proportional to the change in curvature of the base line. The radius of curvature of the base of the petal is given by the expressions

$$r\sqrt{1+\left(\frac{d\phi}{dr}\right)^2} = \frac{r}{k\cdot n} \qquad (18)$$

The increment in plastic energy is given approximately by the expression

$$\frac{2\pi e X'}{N^2} r^2 k\cdot dn$$

before the bourrelet has reached the face of the membrane, but is given qualitatively by the expression

$$\frac{2\pi e X'}{N^2}(a+b-\phi)^2 k\cdot dn$$

after the bourrelet has passed through the face of the membrane. The petals are pushed back until the bourrelet passes the tips of the petals. In the case of an ogival projectile with ogival radius R, the total plastic energy of penetration w is given by the equation

$$w = \frac{\pi e \bar{X}}{2N}\left(\pi a^2 - \frac{\tau}{R}\right) + \frac{2eX'}{N^2}\frac{\tau}{R} + \frac{2\pi e X'}{3N^2}\frac{a^3}{R} \qquad (19)$$

in which τ is the volume within the nose contour from the tip of the projectile to the plane through the edge of the bourrelet. A pointed projectile normally makes a star crack with four petals, for which the average

shear stress $\bar{X}$ is estimated[3] to be $(0.482)X'$. In the case of a 3" AP M79 projectile, the radius R of the ogive is $(3.34)a$, and the volume τ of the ogive is $(1.333)\pi a^3$. The plastic energy for the 3" AP M79 projectile is given by the equation

$$w = (.176)\pi a^2 e X' \tag{20}$$

In the limiting case of a projectile with an infinite ogival radius, the plastic energy would be given by the equation

$$w = (.189)\pi a^2 e X' \tag{21}$$

if the number of petals were still four.

The Motion At Normal Obliquity

The total energy for penetration is the sum of the plastic energy near the impact hole and the elastic energy in the transverse undulation. The effect of the undulation may be calculated with the aid of the substitution

$$\frac{dh}{dp} = \frac{\frac{1}{d}}{1 + \frac{\frac{1}{2}\beta_2 \eta}{h'\sqrt{\beta_1^2 + \beta_2 (h_0' - h')}}} \tag{22}$$

The values of h' in the formula are adjusted by trial until the equation

$$h' = \sqrt{h_0'^2 + 2\int \eta \, dh} \tag{23}$$

reproduces the trial values of h'.

In the case of a 3" AP.M79 projectile in a plate of STS .03 calibers thick the initial value of h' would be .619 in a limit impact.

The Force and Torque at Low Obliquity

The branches of the star crack vary in orientation from impact to impact. If the energy distribution in many impacts were averaged, the average distribution would be relatively smooth, even though the energy per unit area varies in a wide range over each individual petal. The analysis of force and torque at low obliquity is therefore based upon the following assumptions.

(a) It is assumed that the energy per unit perimeter of impact which is required to fold back an annular zone is equivalent to the average energy which would be required in a normal impact.

(b) It is assumed that the energy per unit area of impact which is required to push back the petals is equivalent to the average energy which would be required in a normal impact.

Let r be the perpendicular distance from the axis of the projectile to a point on the rim of the impact, and let φ be the polar angle between the axis i and the perpendicular line to the point on the rim. The polar axis is tilted at the angle χ to the axis k. The equation for an ogive of radius R is then

$$(R - a + r)^2 + \left(b - \frac{p}{\cos\chi} + r\sin\varphi\tan\chi\right)^2 = R^2 \qquad (24)$$

which may be solved with the aid of the quadratic formula to give r as a function of p, φ, χ. Let the unit vector n be normal to the nose contour at the point on the rim. The cartesian components of n are given by the

expressions

$$+\frac{1}{R}(R-a+r)\cos\varphi \qquad \mathbf{i}$$

$$+\frac{1}{R}(R-a+r)\sin\varphi\cos\chi + \frac{1}{R}\left(b - \frac{p}{\cos\chi} + r\sin\varphi\tan\chi\right)\sin\chi \qquad \mathbf{j}$$

$$-\frac{1}{R}(R-a+r)\sin\varphi\sin\chi + \frac{1}{R}\left(b - \frac{p}{\cos\chi} + r\sin\varphi\tan\chi\right)\cos\chi \qquad \mathbf{k}$$

Let the unit vector t be tangent to the nose contour at the point on the rim, and let t be coplanar with the axis of the projectile. The cartesian components of t are given by the expressions

$$+\frac{1}{R}\left(b - \frac{p}{\cos\chi} + r\sin\varphi\tan\chi\right)\cos\varphi \qquad \mathbf{i}$$

$$-\frac{1}{R}(R-a+r)\sin\chi + \frac{1}{R}\left(b - \frac{p}{\cos\chi} + r\sin\varphi\tan\chi\right)\sin\varphi\cos\chi \qquad \mathbf{j}$$

$$-\frac{1}{R}(R-a+r)\cos\chi - \frac{1}{R}\left(b - \frac{p}{\cos\chi} + r\sin\varphi\tan\chi\right)\sin\varphi\sin\chi \qquad \mathbf{k}$$

During an arbitrary displacement δr of the projectile, all points on the nose contour which were located within the vector distance $-\delta r$ from the membrane are brought into contact with the membrane. Of these points, all which lie in the range $d\varphi$ were located in a parallelogram on the nose contour. The base line of the parallelogram is collinear with t and extends to the same distance from the surface of the membrane as the vector $-\delta r$. The base length of the parallelogram is therefore $-(\mathbf{k}\cdot\delta r)/(\mathbf{k}\cdot\mathbf{t})$ and the altitude is $r d\varphi$. This parallelogram on the nose contour is brought

into contact with a parallelogram of area ds on the surface of the membrane. The two parallelograms form two faces of a parallelopiped, while the vector δr is a diagonal of the third face of the same parallelopiped. The volume of the parallelopiped is given by the expressions

$$(\mathrm{k}\cdot\delta r)ds = -\frac{(\mathrm{k}\cdot\delta r)}{(\mathrm{k}\cdot\mathrm{t})}(\mathrm{n}\cdot\delta r)rd\varphi \tag{25}$$

As the surface of the membrane is folded back, the plastic work is given by the expression

$$-\frac{\pi e\bar{X}}{2N}\int\frac{(1-\mathrm{k}\cdot\mathrm{n})}{\mathrm{k}\cdot\mathrm{t}}(\mathrm{n}\cdot\delta r)rd\varphi$$

If the arbitrary displacement δr is parallel to the surface of the membrane, the petals are not pushed back. If the displacement δr is parallel to the axis of the projectile, the plastic work is given approximately by the expression

$$\frac{eX'}{N^2R\cos\chi}\int(\mathrm{k}\cdot\delta r)r^2d\varphi$$

The resultant force f on the projectile is therefore given by the equation

$$\mathrm{f} = +\frac{\pi e\bar{X}}{2N}\int\frac{(1-\mathrm{k}\cdot\mathrm{n})}{\mathrm{k}\cdot\mathrm{t}}\mathrm{n}rd\varphi - \frac{eX'}{N^2R\cos\chi}\mathrm{k}\int r^2d\varphi \tag{26}$$

The line of action of any element of force on a frictionless ogive coincides with n and passes through the center of curvature of the ogive. The loci of the centers of curvature lie on a plane which passes through the leading edge of the bourrelet. The element of force applies a torque about

a central axis which lies in this plane at the bourrelet. The lever arm of the torque is given by the expressions

$$\frac{1}{R}(R-a)\left(b - \frac{p}{\cos\chi} + r\sin\varphi\tan\chi\right) = -\,(R-a)(\mathrm{k}\cdot\mathrm{t})\frac{\partial r}{\partial\phi} \tag{27}$$

The same torque would be applied if the element of force coincident with n were replaced by the reverse of its component along the axis of the projectile, and if the lever arm were replaced by $R-a$. To the torque about the axis at the bourrelet must be added the torque between the bourrelet and the center of mass. The resultant torque L with respect to the center of mass is given by the equation

$$\begin{aligned} L = &-\lambda(\mathrm{j}\cos\chi - \mathrm{k}\sin\chi)\cdot\mathrm{f} \\ &+ \frac{\pi e\bar{X}}{2N}(R-a)\int(1-\mathrm{k}\cdot\mathrm{n})\,r\frac{\partial r}{\partial\phi}\sin\varphi d\varphi \\ &+ \frac{eX'}{N^2R}(R-a)\int r^2\sin\varphi d\varphi \end{aligned} \tag{28}$$

The limits of integration with respect to φ depend upon that component of the local velocity which is normal to the surface of the projectile. Wherever the normal component of the local velocity is negative, the projectile loses contact with the membrane. During the first stage of the penetration, the normal component of local velocity is everywhere positive for any velocity of the center of mass which is in a range of velocities. The limits of integration with respect to φ are 0 to 2π, and the force and torque are independent of velocity. The normal component

of local velocity is given by the expression

$$+\frac{1}{R}(R-a+r)(\dot{y}\cos\chi - (\dot{z}-\dot{\xi})\sin\chi + \lambda\dot{\chi})\sin\varphi$$

$$+\frac{1}{R}(b - \frac{p}{\cos\chi} + r\sin\varphi\tan\chi)(\dot{y}\sin\chi + (\dot{z}-\dot{\xi})\cos\chi + (R-a)\dot{\chi}\sin\varphi)$$

After the projectile has lost contact with the membrane, it does not renew the contact until the coordinates

$$r\cos\varphi \text{ and } y-(z-\xi)\tan\chi + r\frac{\sin\varphi}{\cos\chi}$$

return to the last values which they had before loss of contact.

The theoretical forces and torque on a 3" AP M79 projectile with a stationary axis are listed for a few obliquities in Tables I to V. The ratio p/d in each table is zero when the tip of the projectile touches the membrane. The side of the nose contour touches first if the obliquity is 45° or greater, and the ratio p/d is initially negative at high obliquity. The penetrations when the edge of the bourrelet is in contact with the membrane are bracketed by asterisks. The table ends at that penetration where the edge of the bourrelet passes the tips of the petals. The effect of partial contact between membrane and projectile is illustrated by the entries in each table for different values of θ, which differ from each value of χ by the infinitesimal amounts $\pm\epsilon$, or by the angles $\pm\frac{1}{4}\pi$

The theoretical curve for the force at $\chi = 0$ is compared in Figure (2) with an experimental curve, which has been derived from the depth of pene-

tration of projectiles into armor The experimental curve for $e/d = .03$ is based on experimental data at $e/d = .25$, and some of the discrepancy between the theoretical and experimental curves may possibly be caused by the plastic flow which is required to bring the thicker plate to the point of fracture.

The analysis of force and torque has been guided by some experiments with an ogival indenter in aluminum foil. The indenter was turned out of wood and the foil was stretched over the mouth of a glass tumbler. The experiment was performed on the weighing pan of a chemical platform scales. Two types of indentation were made at an inclination of 35°.

In the first type of indentation a force was applied parallel to the axis of the indenter. The component of force normal to the foil had a maximum value of 50 grams weight. The edge of the hole in the foil was tangent to a transverse scratch, which passed through the tip of the projectile at first contact.

In the second type of indentation the indenter was mounted in a heavy clamp and the clamp was suspended by a thread. The tip of the projectile, the center of mass of the suspended assembly, and the point of support were adjusted in the same vertical line. As the indenter was lowered onto the foil, the force was applied by gravity normal to the foil. The maximum force normal to the foil was 71 grams weight. The edge of the hole in the foil was also tangent to a transverse scratch, which passed through the tip of the projectile at first contact.

The two holes were, however, on opposite sides of the scratch. This is interpreted as evidence that the force for complete contact lies between the axis of the indenter and the normal of the foil. When any other force is applied, the zone of contact is limited automatically.

The component of force along the axis of the indenter had a maximum value of 61 grams weight in the first test, and had a maximum value of 58 grams weight in the second test. The component of force along the axis of the indenter was therefore relatively insensitive to the direction of force.

The Motion at Low Obliquity

In the limiting case of an impact at a velocity far above limit, the path of the projectile deviates relatively little from a straight line. The rotation of the velocity of the center of mass and the rotation of the axis of the projectile tend to vary inversely with the square of the striking velocity at any particular stage of the penetration, whereas the displacement of the undulation tends to vary inversely as the striking velocity. The motion in the undulation causes the projectile to lose contact with the membrane in the neighborhood of $\varphi = -\frac{1}{2}\pi$ when the cylindrical body of the projectile reaches the membrane.

In the limiting case of an impact at a velocity just at limit, the rotation of the velocity of the center of mass, and the rotation of the axis of the projectile are relatively large. A numerical integration of the equations of motion has been completed for the special case of an impact by a 3" AP M79 projectile at 45° obliquity. On the basis of this computation, it is estimated that the projectile loses contact with the membrane, in the neighborhood of $\varphi = -\frac{1}{2}\pi$, before the bourrelet reaches the membrane. The projectile travels a quarter of a caliber, after this loss of contact, before the bourrelet first reaches the membrane at $\varphi = -\frac{1}{2}\pi$. The projectile then loses contact in the neighborhood of $\varphi = +\frac{1}{2}\pi$, after the projectile has traveled a further distance of a quarter of a caliber, and before the bourrelet has reached the membrane at $\varphi = +\frac{1}{2}\pi$. The angles θ and χ are not very different at the instant when contact is lost on both

sides of the nose of the projectile, but the axis of the projectile retains a relatively large angular velocity. The petals of the membrane continue to apply a force on the nose of the projectile after the partial loss of contact. When the axis of the projectile is nearly orthogonal to the membrane, contact is restored at $\varphi = +\frac{1}{2}\pi$, but then the residual velocity of the center of mass is relatively small, and the final velocity is quickly deflected toward the normal of the membrane.

The theoretical motion of the projectile is illustrated by Figure (3). The rotation of the axis of the projectile has been recorded experimentally by flash x-radiograms of projectiles in flight, and the final deflection of the velocity is supported by many measurements of deflection at the Armor and Projectile Laboratory.

On the basis of the computations for 45° obliquity, the subsequent analysis for other obliquities has been simplified by the introduction of the following simplifications.

(a) It is assumed that the projectile remains in complete contact with the membrane until the bourrelet first touches the membrane.

(b) It is assumed that a transverse force brings the velocity of the center of mass instantaneously into coincidence with the axis of the projectile when the bourrelet first touches the membrane.

(c) It is assumed that the projectile continues without transverse force or torque after the bourrelet has first touched the membrane. It is assumed that the longitudinal force on the actual projectile with a rotating axis is the same as the longitudinal force on a

projectile with a stationary axis.

(d) It is assumed that the final deflection of the velocity of the center of mass occurs too late in the penetration to be of influence on the limit velocity for complete penetration.

(e) It is assumed that the motion in the undulation is the same as the motion in an elastic undulation with the force concentrated at a point.

Stage (i). During the first stage, the variables g', h' have been calculated by the equations

$$g' = g_0' + \int\left(\frac{\gamma}{h'}\right)dh \tag{29}$$

$$h' = \sqrt{h_0'^2 + 2\int\eta\, dh} \tag{30}$$

The variable ϕ was used as the variable of integration in accordance with the substitution

$$\frac{dh}{d\phi} = \frac{\frac{1}{d}}{1 + \frac{\frac{1}{2}\beta_2\eta}{h'\sqrt{\beta_1^2 + \beta_2(h_0' - h')}}} \tag{31}$$

At the end of the first stage, the variables g', h' were transformed into the initial values g_0', h_0' for the next stage by the substitutions

$$g' - (g'\cos\chi - h'\sin\chi)\cos\chi \rightarrow g_0'$$

$$h' + (g'\cos\chi - h'\sin\chi)\sin\chi \rightarrow h_0'$$

Stage (ii). During the second stage, the initial value of h' was made to satisfy the equation

$$h_0' = \sqrt{-2\int \eta dh} \tag{32}$$

for which η was the component, normal to the membrane, of a longitudinal vector of magnitude $\eta\cos\chi + \nu\sin\chi$.

Stage (iii). Sidewise motion of the projectile during a limit impact was without influence on the calculations.

The variation of h' with ϕ was adjusted by trial until the equations were satisfied. A sample calculation has been completed for the case of a 3" AP M79 projectile in a membrane of STS with a thickness of .03 calibers and a static tensile strength of 115000(lb)/(in)^2. The yield stress X' was assumed to be equal to the dynamic tensile strength, which is 145000 (lb)/(in)^2. The results are summarized in Table VII, and are consistent with the equation

$$\frac{dh_0'}{d\cos\theta_0} = .252 \qquad (\chi_0 \equiv \theta_0) \tag{33}$$

The theoretical plate penetration function is compared with the standard ballistic function in Figure (5).

The Force and Torque at High Obliquity

At the beginning of the first stage of the penetration, an ogival projectile makes a shallow depression in a thin membrane. During the first instant of contact, the rim of the depression spreads out at a greater velocity than the maximum velocity of propagation for an undulation in the membrane. Motion of the projectile in a direction normal to the membrane sets the membrane into motion with a velocity which is symmetrically distributed over the area of contact. Motion of the projectile in a direction parallel to the membrane increases the velocity of the membrane near the leading edge of the depression, and decreases the velocity near the trailing edge. This increment in velocity, which is associated with motion parallel to the membrane, is antisymmetrically distributed over the area of contact. The symmetric distribution of velocity is the major contributor to the total momentum of the membrane, since the antisymmetric distribution tends to average out. The total momentum in the membrane during the first instant of contact is therefore essentially independent of motion of the projectile parallel to the membrane as long as the critical velocity for cavitation is not exceeded.

The force on the projectile jumps suddenly on contact to a finite force. The initial pressure on the projectile consists of two parts, a transient dynamic pressure which is concentrated along the rim of the depression, and a steady plastic pressure which is uniform over the area of contact.

The transient dynamic pressure is initially equal to the expression

$$\rho \dot{s}^2$$

The steady plastic pressure is sufficient to prevent cavitation unless it is exceeded by the steady Bernoullian pressure, whose magnitude is given by the expression

$$\frac{e}{R}\rho v^2$$

The length of the depression is greater than the width and one principal radius of curvature is greater than the other. The larger radius of curvature is equal to the ogival radius R of the nose contour, while the smaller radius of curvature is given by the expression

$$R - \frac{R-a}{\sin\chi}$$

The steady plastic pressure is given approximately by the expression

$$\left(\frac{1}{R} + \frac{1}{R - \frac{R-a}{\sin\chi}}\right)eX'$$

in the limiting case of a thin membrane. The force on the projectile gradually subsides until an undulation begins to form.

When the velocity of the rim of the depression has subsided to the velocity of propagation, the rim detaches itself from the surface of the projectile and continues outward with the velocity of propagation. The formation of an undulation begins near the trailing edge of the depression, where the membrane is moving with the least velocity. The undulation may eventually surround the projectile if the projectile is moving slowly, but

can never surround a projectile whose velocity parallel to the membrane is greater than the velocity of propagation. The total momentum in the membrane is given approximately by the expression

$$\rho e \dot{z} s'$$

in which s' is the effective area of the depression. The area s' is equal to the sum of the total area of contact and a fraction of the free area of the undulation.

The force on the projectile increases gradually, after the undulation begins to develop, until the membrane fractures. The projectile moves a limited distance from the instant of contact with the membrane to the instant of fracture of the membrane, but the force on the projectile at the instant of fracture is nearly proportional to $\dot{z}$.

The membrane probably thins down to fracture in a locallized zone when the projectile is moving slowly, but local necking or webbing do not have time to develop when the projectile is moving rapidly.

After the membrane has cracked it offers less resistance to the projectile. The principal axis of stress at the free edge of the crack is parallel to the edge. Any line element which is perpendicular to the crack before fracture continues to be perpendicular after fracture. The two components of stress perpendicular to the edge are zero in the limiting case of a thin membrane, and the stress is a pure tension or compression parallel to the edge. The plastic pressure on the nose of the projectile just after fracture is therefore given approximately by the expression

$$\frac{e}{R} X'$$

The motion of the undulation is retarded just after fracture, and the penetration of the projectile is accelerated, until the resistance to the projectile is restored.

The edges of the crack can apply relatively little pressure to the parallel sides of the projectile and the center of pressure is forward of the bourrelet. The axis of the projectile is rotated from its original orientation and the nose of the projectile is displaced from its original line of flight. The base continues with little deviation from its original line of flight. The projectile loses contact with the membrane just forward of the bourrelet, but renews the contact just forward of the base. The distances λ_1, λ_2 from the center of mass to the points of contact are related approximately by the equation

$$\lambda_1 \lambda_2 = \frac{1}{m} C \tag{34}$$

The base scrapes the membrane, after renewed contact, into an elliptic cylindrical surface whose axis is in the same direction as the original obliquity. The membrane applies to the base a pressure which is concentrated along the rim of the base.

When the projectile has turned into a position with its axis nearly coplanar with the membrane, the force on the nose is given approximately by the expression

$$-\frac{a^2}{R}\,\mathrm{j}\,eX' - \{R\cos^{-1}(1-\frac{a}{R}) - b(1-\frac{a}{R})\}\{1-\cos(1)\}\mathrm{k}\,eX'$$

and the torque on the nose is given approximately by the expression

$$-a^2(1-\frac{a}{R})\{1-\cos(1)\}eX' - \lambda\mathrm{k}\cdot f$$

The tension in the membrane at the rim of the base is perpendicular to the rim and is tangent to the cylindrical segments of the membrane in the rear of the projectile. If the axis of the cylindrical segments is inclined at the angle θ to the normal of the plate, then the resultant force on the base is given approximately by the expression

$$- 2ake X' \int_0^1 \frac{\cos\theta \sin^2\varphi d\varphi}{\sqrt{1 - \cos^2\theta \cos^2\varphi}}$$

and the torque on the base is given approximately by the expression

$$+ 2a(l - b - \lambda) e X' \int_0^1 \frac{\cos\theta \sin^2\varphi d\varphi}{\sqrt{1 - \cos^2\theta \cos^2\varphi}}$$

in which l is the length of the projectile. The integral is expressed in terms of elliptic integrals by the equation

$$\int_0^1 \frac{\cos\theta \sin^2\varphi d\varphi}{\sqrt{1 - \cos^2\theta \cos^2\varphi}} = \frac{1}{k}(E(k, \frac{\pi}{2}) - E(k, \frac{\pi}{2} - 1)) - \frac{1-k^2}{k}(F(k, \frac{\pi}{2}) - F(k, \frac{\pi}{2} - 1))$$

for which the modulus k is given by the equation

$$k = \cos\theta$$

During a limit impact on a thin membrane, the force on an ogival projectile at the instant when the projectile first makes contact with the membrane, at the instant when the projectile fractures the membrane, and at the instant when the projectile is coplanar with the membrane, all happen to be of the same magnitude as the average force on the projectile.

The analysis of force and torque on a projectile during the first two stages is therefore based upon the following simplifications.

(a) It is assumed that the force and torque on the nose of the projectile are constant and are equal to the force and torque on a projectile whose axis is coplanar with the membrane.

(b) It is assumed that the force and torque on the base of the projectile are constant, after the base reaches the membrane, and are equal to the force and torque on the base of a projectile whose axis is coplanar with the membrane.

(c) It is assumed that the displacement of the undulation is the same as the displacement in an elastic undulation with the force concentrated at a point.

(d) It is assumed that the second stage comes to an end when the ratio of the torque to the force is essentially the same for the second and third stages.

During the third stage of the penetration, the projectile moves sidewise through the membrane. The force on the projectile is then nearly independent of the position of the center of mass as long as the nose or base of the projectile is not even with the membrane. The torque on the projectile is, however, a function of the position of the center of mass.

Of the total surface of the membrane which has been displaced by the projectile at any instant, that part at the extreme forward end has, before displacement, the shape of two quadrants of a plane ellipse with the crack for a common radius, and has, after displacement, the shape of two segments

of a cylinder, with the ellipse for a base. The transformation of a quadrant of the ellipse into a segment of the cylinder requires the same strain as would be required by the distortion of a plane quadrant into a new plane surface which has the same final contour as the segment would have if it were rolled out flat.

If χ is the angle between the axis of the projectile and the normal to the membrane, then the arc of the ellipse is represented analytically by the parametric equations

$$x = a\cos\varphi \qquad y = \frac{a\sin\varphi}{\cos\chi} \tag{35}$$

in which x, y are the cartesian coordinates in the plane of the ellipse and φ is a cylindrical polar coordinate. The base of the segment is represented analytically by the parametric equations

$$x' = a\varphi \qquad y' = a\tan\chi\sin\varphi \tag{36}$$

in which x', y' are the cartesian coordinates in the plane of the segment after flattening, and φ is the same polar coordinate.

Let a straight line be considered which is perpendicular to the crack before transformation. The length of the line from the crack to the arc of the quadrant is $a\cos\varphi$. The line intersects the arc of the quadrant at an angle which is given by the equation

$$-\tan^{-1}\frac{dy}{dx} = +\tan^{-1}\frac{\cot\varphi}{\cos\chi} \tag{37}$$

Two such lines, whose values of φ differ by the differential $d\varphi$, are separated by the distance

$$\frac{a\cos\varphi}{\cos\chi}\,d\varphi$$

The angles which the two lines make with the arc of the quadrant differ by the amount

$$-\,d\tan^{-1}\frac{dy}{dx} = -\,\frac{\cos\chi}{\cos^2\varphi + \cos^2\chi\sin^2\varphi}\,d\varphi \qquad (38)$$

Let a straight line be considered, which is tangent to the base of the segment. This line is oriented at an angle which is given by the equation

$$\tan^{-1}\frac{dy'}{dx'} = \tan^{-1}(\tan\chi\cos\varphi) \qquad (39)$$

Two such lines whose values of φ differ by $d\varphi$, are inclined to each other with the angle

$$d\tan^{-1}\frac{dy'}{dx'} = -\,\frac{\tan\chi\sin\varphi}{1 + \tan^2\chi\cos^2\varphi}\,d\varphi \qquad (40)$$

Let a straight line be considered, which makes the same angle with the base of the segment, as the original straight line, perpendicular to the crack at the same value of φ, made with the arc of the quadrant. This new line is then oriented at an angle which is given by the expression

$$\tan^{-1}\frac{dy}{dx} + \tan^{-1}\frac{dy'}{dx'}$$

Two such lines whose values of φ differ by the differential $d\varphi$ are

inclined to each other at the angle

$$\frac{\cos\chi}{1 + \sin\chi\sin\varphi}\, d\varphi$$

If the length of these lines is $a\cos\varphi$, then the termini of the lines are the loci of a curve which is a good approximation for the contour into which the edge of the crack is fanned out by the transformation. The length of the contour is given by the equation

$$\int_0^{\frac{\pi}{2}} \frac{a\cos\varphi d\varphi}{\cos\chi} + \int_0^{\frac{\pi}{2}} \frac{a\cos\chi\cos\varphi}{1 + \sin\chi\sin\varphi}\, d\varphi = \frac{a}{\cos\chi} + a\frac{\cos\chi}{\sin\chi} \log(1 + \sin\chi) \qquad (41)$$

The length of the crack is therefore greater than its original length by the amount

$$a\frac{\cos\chi}{\sin\chi} \log(1 + \sin\chi)$$

while the length of the arc or base is unchanged. The elongation of any line which is parallel to the edge of the crack is proportional to its distance from the arc or base. The total energy w which is required by the transformation is therefore given approximately by the equation

$$w = a^2 \delta X' \frac{\cos\chi}{\sin\chi} \log(1 + \sin\chi) \qquad (42)$$

If the sides of a long crack were folded down uniformly along a straight line, the strain would be zero. The strain at the edge of the actual crack tends, therefore, to return to zero after the projectile has passed by, or the edge of the crack tends to curl up. During a steady motion of the projectile, the length of the crack would steadily increase.

The final strain at the edge of the crack is approximately equal to $\cos^2\chi$. The plastic energy per unit length of crack would therefore be $a\epsilon X'\cos^2\chi$. This is also the component of force on the projectile in the direction parallel to the surface of the membrane. The force itself is perpendicular to the axis of the projectile, and is given by the equation

$$f = -(j\cos\chi - k\sin\chi)a\epsilon X'\cos\chi \qquad (43)$$

The torque on the projectile is given approximately by the equation

$$L = -(z - \xi + \frac{\pi}{4}a\sin\chi)a\epsilon X' \qquad (44)$$

These equations are valid when χ is in a range which does not include $\frac{1}{2}\pi$ or π, and give a force and torque which is too small when χ is equal to $\frac{1}{2}\pi$ or π.

During a limit impact the projectile see-saws back and forth in the membrane and the angle χ is limited to a range which straddles π. The displacement of the projectile tends to lead the displacement of the undulation by a phase angle of $\frac{1}{2}\pi$, and the motion of the projectile is damped by the undulation, but the amplitude of the undulation in the third stage is limited by the tension which is built up in the first two stages. The analysis of force and torque on a projectile during the third stage is therefore based upon the following simplifications.

(a) It is assumed that the force and torque are given by the limiting functions when χ is nearly equal to π.

(b) It is assumed that the displacement of the undulation is stationary.

(c) It is assumed that the third stage comes to an end when the projectile loses contact with the membrane.

The Motion at High Obliquity

Stage (i). The initial values of h_0 and χ_0 in the first stage are related in accordance with the equation

$$h_0 = -\frac{1}{d}(R - (R-a)\sin\chi_0 + \lambda\cos\chi_0) \tag{45}$$

The initial values of g_0, h_0, χ_0, g_0', h_0', χ_0' are otherwise arbitrary. The assumed values of γ, η, Λ are constants, and independent of obliquity. The instantaneous values of g, h, χ, g', h', χ' have been calculated with the aid of equations (9), (10), (11) and the instantaneous values of ξ/d were calculated with the aid of Equation (13). The first stage comes to an end when the base of the projectile reaches the membrane, or when h satisfies the equation

$$h = \frac{\xi}{d} + \frac{1}{d}((l-b-\lambda)\cos\chi - a\sin\chi) \tag{46}$$

Inasmuch as h and χ are quadratic functions of q, the final values of g, h, χ, g', h', χ' were found by a method of successive approximations.

Stage (ii). The initial values of g_0, h_0, χ_0, g_0', h_0', χ_0' in the second stage are the final values of g, h, χ, g', h', χ' in the first stage. The calculated values of γ, η, Λ are constants, but depend upon the initial obliquity. The instantaneous values of g, h, χ, g', h', χ' have been calculated with the aid of Equations (9), (10), (11). The instantaneous values of ξ/d were calculated with the aid of Equation (13), but with h_0' in the equation replaced by the initial value of h_0' in the first stage.

The second stage comes to an end when the angle χ is approximately equal to

$$\pi - \cos^{-1}\left(\frac{\pi}{8}\frac{\eta}{\Lambda}\right)$$

Inasmuch as χ is a quadratic function of q the final value of q was found with the aid of the quadratic formula.

Stage (iii). The initial values of g_0, h_0, χ_0, g_0', h_0', χ_0' in the third stage are the final values of g, h, χ, g', h', χ' in the second stage. Insofar as the difference $\chi-\pi$ is small but finite, the functions γ, η, Λ may be expressed by the limiting equations

$$\gamma = -\frac{1}{2} \qquad \eta = +\frac{1}{2}(\chi - \pi) \tag{47}$$

$$\Lambda = \frac{1}{2}\left(h - \frac{\xi}{d}\right) - \frac{\pi}{16}(\chi - \pi) \tag{48}$$

The general integrals of the equations of motion are then

$$g = g_0 + g_0' q - \frac{1}{4}q^2 \tag{49}$$

$$h - \frac{\xi}{d} = + h_1 \sinh\mu_1 q + h_2 \cosh\mu_1 q + h_3 \sin\mu_2 q + h_4 \cos\mu_2 q \tag{50}$$

$$\chi - \pi = + 2\mu_1^2 h_1 \sinh\mu_1 q + 2\mu_1^2 h_2 \cosh\mu_1 q - 2\mu_2^2 h_3 \sin\mu_2 q - 2\mu_2^2 h_4 \cos\mu_2 q \tag{51}$$

in which μ_1, μ_2, h_1, h_2, h_3, h_4 are constants of integration. The constants μ_1 and μ_2 are given by the equations

$$\mu_1 = \left(\frac{1}{2}\sqrt{\alpha + \frac{\pi^2}{256}\alpha^2} - \frac{\pi}{32}\alpha\right)^{\frac{1}{2}} \qquad \mu_2 = \left(\frac{1}{2}\sqrt{\alpha + \frac{\pi^2}{256}\alpha^2} + \frac{\pi}{32}\alpha\right)^{\frac{1}{2}} \tag{52}$$

The constants h_1, h_2, h_3, h_4 are given in terms of the constants h_0, χ_0, h_0', χ_0' by the equations

$$
\begin{aligned}
h_1 &= \frac{2\mu_2^2 h_0' + \chi_0'}{2\mu_1(\mu_1^2 + \mu_2^2)} \\
h_2 &= \frac{2\mu_2^2(h_0 - \frac{\xi}{d}) + (\chi_0 - \pi)}{2(\mu_1^2 + \mu_2^2)} \\
h_3 &= \frac{2\mu_1^2 h_0' - \chi_0'}{2\mu_2(\mu_1^2 + \mu_2^2)} \\
h_4 &= \frac{2\mu_1^2(h_0 - \frac{\xi}{d}) - (\chi_0 - \pi)}{2(\mu_1^2 + \mu_2^2)}
\end{aligned}
\tag{53}
$$

The projectile continues to see-saw in the plate until $g \leq 0$.
The projectile may remain briefly in contact with the membrane after the plastic deformation of the membrane has ceased. The final value of q is therefore greater than $2g_0'$. The impact is a limit impact if $h' = 0$ and $\chi' = 0$ at the end of contact. The approximate conditions for a limit impact are therefore

$$h_1 \cosh \mu_1 q + h_2 \sinh \mu_1 q = 0 \tag{54}$$

$$h_3 \cos \mu_2 q - h_4 \sin \mu_2 q = 0 \tag{55}$$

which are strictly correct when g', h', χ' happen to vanish simultaneously.

The initial values of g_0', h_0' at the beginning of the first stage were adjusted by trial until the conditions for a limit impact were satisfied at the end of the third stage.

The theoretical motion of the projectile during a limit impact at 75° obliquity is illustrated by Figure (4).

A sample calculation has been completed for the case of a 3" AP M79 projectile in a membrane of STS with a thickness of .03 calibers and a static tensile strength of 115000(lb)/(in)^2. The yield stress X' was assumed to be equal to the dynamic tensile strength, which is at least 145000 (lb)/(in)^2. The results are summarized in Tables VI and VII. The theoretical plate penetration function is compared with the standard ballistic function in Figure (5).

Cavitation at High Obliquity

There is a lower critical obliquity above which the plastic stress is unable to prevent cavitation, and the dynamic stress contributes to the resistance to penetration.

In the limiting case of a hypervelocity projectile with its axis parallel to the membrane, the work which is required to displace the axis of the projectile into coincidence with the membrane against the dynamic force is given by the expression

$$\frac{1}{4}\rho v^2 eR^2\left(\cos^{-1}\frac{(R-a)}{R} - \frac{(R-a)b}{R^2} - \frac{2}{3}\frac{(R-a)b^3}{R^4}\right)$$

The dynamic work varies as the square of the velocity, whereas the plastic work is essentially constant. Associated with this work is the kinetic energy

$$\frac{1}{2}mv^2\cos^2\theta$$

which also varies as the square of the velocity. The kinetic energy cannot therefore exceed the dynamic work if the obliquity θ exceeds an upper critical obliquity, and a complete penetration is then impossible.

In the case of a 3" AP M79 projectile, the estimated lower obliquity for cavitation is given by the expression

$$\cot^{-1}\left((.271)\frac{e}{d}\right)^{\frac{1}{2}}$$

while the estimated upper obliquity for a complete penetration is given by

the expression

$$\cot^{-1}\{(.088)\frac{e}{d}\}^{\frac{1}{2}}$$

The projectile would lose less than four percent of its velocity during a ricochet at the upper limit of obliquity in a membrane with a thickness of .03 calibers. The critical obliquities for this case are given in Table VII.

Ballistic Tests

The thicknesses of STS plates have been measured after impact by 5" Comm Mk 46-2 projectiles at 70° obliquity. The average minimum thickness of the plates, where the fractures originated, was 65% of the initial thickness. The shear strain function at fracture was therefore at least .28.

A few ballistic tests have been completed with uncapped 37 mm AP M51B2 projectiles at 75° obliquity in a 0.25" plate of STS. Fracture of the plate occurred at a greater depth than the theoretical limit for a thin membrane. A finite additional amount of energy was required to bring the plate to the point of fracture. The projectile blasted an opening in the plate which was two calibers wide, and cavitation may therefore have been present during penetration. That the projectile actually moves sidewise through the plate at the end of a near-limit impact is proved by flash X-radiograms, one of which is reproduced in Figure (6). The final appearance of the impact hole is illustrated by Figure (7). An attempt to obtain an unquestionably complete penetration was unfortunately thwarted by projectile breakage. The fiber stress in the projectile is estimated to be 80000 $(lb)/(in)^2$ when the projectile moves sidewise in a plate .25" thick, whereas fracture occurs in the projectile steel at a fiber stress of 325000 $(lb)/(in)^2$. Examination of flash X-radiograms of the breaking projectiles shows that fracture occurred at the forward edge of the bandscore where there is evidently a notch concentration of stress.

THE EFFECTS OF YAW AND OF RATE OF YAW

Low Obliquity

The effect on h_0' of a small increment in θ_0 at low obliquity was calculated with χ_0 held constant, in order to obtain an estimate of $\partial h_0'/\partial\theta_0$. The magnitude of $\partial h_0'/\partial\theta_0$ was then estimated with the aid of the relationship

$$\frac{dh_0'}{d\theta_0} = \frac{\partial h_0'}{\partial \partial_0} + \frac{\partial h_0'}{\partial \chi_0} \qquad (\theta_0 \equiv \chi_0) \qquad (56)$$

The increment $\Delta\chi$ which is associated with an initial rate of yaw χ_0' is given approximately by the equation*

$$\Delta\chi = \chi_0' q \qquad (57)$$

The effect on h_0' of this increment in χ was estimated with the aid of the formula

$$\Delta\int \eta dh = \chi_0' \int \frac{\partial}{\partial\chi}(\eta\frac{dh}{d\phi}) q d\phi \qquad (58)$$

The results of calculation are given in Table VII.

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* An inspection of Equation (10) has shown that the average error in this approximation is of the order of 10%.

High Obliquity

Insofar as the theory is a valid representation of the force and torque at high obliquity, the effect on h_0' of small increments in χ_0 or χ_0' may be determined by direct calculations with various initial values of θ_0, χ_0, χ_0'. The results of such calculations are summarized in Table VII.

Ballistic Tests

The effect of yaw on the limit energy has only been determined with certainty by ballistic tests with projectiles against thick plates at normal obliquity. The effect on the limit energy of one degree of yaw was found to be equivalent to the effect of two degrees of obliquity. The theoretical effect on the limit energy of one degree of yaw is only a fraction of the effect of one degree of obliquity in the case of a thin membrane.

APPENDIX A

TABLE I

The theoretical forces and torque on a 3" AP M79 projectile in a thin membrane, at $\chi = 0$ and $\chi' = 0$

$\theta = 0 \pm \varepsilon$

$\frac{p}{d}$	Υ	η	Λ
.000	.000	.000	.000
.149	000	-.052	000
.298	000	-.098	000
.447	000	-.131	000
.596	000	-.150	000
.894	.000	-.136	000
*1.192	.000	-.059	.000
**1.192	∓.189	-.059	∓.107
1.317	∓.189	-.033	∓.083
1.442	∓.189	-.015	∓.059
1.567	∓.189	-.004	∓.035
1.692	∓.189	.000	∓.012

* Edge of bourrelet approaches membrane.
** Edge of bourrelet passes membrane.

TABLE II

The theoretical forces and torque on a 3" AP M79 projectile in a thin membrane at $\chi = 15°$ and $\chi' = 0$

$\frac{p}{d}$	$\theta = \chi + \varepsilon$			$\theta = \chi - \varepsilon$		
	γ	η	Λ	γ	η	Λ
000	000	.000	000	000	000	000
.255	-.024	-.089	-.004	-.024	-.089	-.004
.511	-.031	-.142	-.013	-.031	-.142	-.013
.766	-.049	-.144	-.027	-.049	-.144	-.027
*1.022	-.053	-.094	-.043	-.053	-.094	-.043
1.086	-.186	-.039	-.122	-.058	-.074	-.054
1.151	-.216	-.012	-.140	-.065	-.052	-.068
1.216	-.223	+.009	-.141	-.045	-.039	-.064
**1.280	-.228	+.026	-.134	+.151	-.076	+.043
1.530	-.228	+.053	-.068	+.151	-.048	+.007
1.780	-.228	+.061	-.004	+.151	-.040	-.030

* Edge of bourrelet reaches membrane at $\varphi = -\frac{1}{2}\pi$
** Edge of bourrelet reaches membrane at $\varphi = +\frac{1}{2}\pi$

TABLE III

The theoretical forces and torque on a 3" AP 79 projectile in a thin membrane at $\chi = 30°$ and $\chi' = 0$

$\frac{p}{d}$	$\theta = \chi + \varepsilon$			$\theta = \chi - \varepsilon$		
	Υ	η	Λ	Υ	η	Λ
.000	.000	.000	.000	000	000	000
.196	-.041	-.081	-.002	-.041	-.081	-.002
.391	-.069	-.130	-.015	-.069	-.130	-.015
.586	-.087	-.141	-.039	-.087	-.141	-.039
*.782	-.078	-.115	-.052	-.078	-.115	-.052
.907	-.204	-.020	-.152	-.110	-.074	-.080
1.032	-.241	+.040	-.180	-.126	-.026	-.134
1.157	-.255	+.089	-.188	-.110	+.006	-.139
**1.282	-.264	+.127	-.164	+.115	-.092	-.002
1.532	-.264	+.144	-.068	+.115	-.075	-.034
1.782	-.264	+.152	+.022	+.115	-.066	-.069

* Edge of bourrelet reaches membrane at $\varphi = -\frac{1}{2}\pi$

** Edge of bourrelet reaches membrane at $\varphi = +\frac{1}{2}\pi$

TABLE IV

The theoretical forces and torque on a 3" AP M79 projectile in a thin membrane at $\chi = 45°$ and $\chi' = 0$

$\frac{\phi}{d}$	$\theta = \chi + \frac{1}{2}\pi$			$\theta = \chi + \varepsilon$			$\theta = \chi - \varepsilon$			$\theta = \chi - \frac{1}{2}\pi$		
	Υ	η	Λ	Υ	η	Λ	Υ	η	Λ	Υ	η	Λ
-7×10^{-5}	000	000	000	000	.000	.000	000	000	000	000	000	000
000	---	---	---	-4×10^{-8}	-5×10^{-4}	$+6\times10^{-4}$	-4×10^{-8}	-5×10^{-4}	$+6\times10^{-4}$	---	---	---
.061	-.026	-.012	-.017	-.030	-.055	+.020	-.030	-.055	+.020	-.004	-.050	+.045
.122	-.051	-.024	-.032	-.051	-.086	+.020	-.051	-.086	+.020	.000	-.077	+.066
.245	-.098	-.040	-.064	-.083	-.125	+.004	-.083	-.125	+.004	+.015	-.116	+.093
.367	-.141	-.044	-.097	-.106	-.139	-.024	-.106	-.139	-.024	+.035	-.141	+.105
*.489	-.180	-.034	-.123	-.123	-.131	-.058	-.123	-.131	-.058	+.057	-.155	+.107
.666	-.227	+.014	-.184	-.215	-.010	-.169	-.151	-.075	-.132	+.076	-.148	+.076
.843	-.259	+.094	-.228	-.259	+.094	-.228	-.175	+.010	-.203	+.084	-.135	+.032
1.019	-.282	+.191	-.242	-.282	+.191	-.242	-.167	+.076	-.226	+.084	-.124	-.009
** 196	-.294	+.269	-.201	-.294	+.269	-.201	+.084	-.109	-.041	+.084	-.109	-.041
1.343	-.294	+.284	-.111	-.294	+.284	-.111	+.084	-.095	-.062	+.084	-.095	-.062
1.596	-.294	+.294	+.100	-.294	+.294	+.100	+.084	-.084	-.119	+.084	-.084	-.119

* Edge of bourrelet reaches membrane at $\varphi = -\frac{1}{2}\pi$

** Edge of bourrelet reaches membrane at $\varphi = +\frac{1}{2}\pi$

TABLE V

The theoretical forces and torque on a 3" AP M79 projectile in a thin membrane at $\chi = 60°$ and $\chi' = 0$

$\frac{p}{d}$	$\theta = \chi + \varepsilon$			$\theta = \chi - \varepsilon$		
	Υ	η	Λ	Υ	η	Λ
-.061	000	000	000	000	000	000
-.020	-.003	-.061	+.064	-.003	-.061	+.064
.000	-.015	-.112	+.109	-.015	-.112	+.109
.041	-.041	-.143	+.101	-.041	-.143	+.101
.081	-.064	-.152	+.078	-.064	-.152	+.078
.122	-.084	-.160	+.056	-.084	-.160	+.056
*.163	-.101	-.163	+.034	-.101	-.163	+.034
.379	-.214	-.029	-.143	-.171	-.102	-.117
.596	-.265	+.133	-.270	-.204	+.028	-.270
.812	-.304	+.326	-.331	-.211	+.165	-.356
**1.029	-.318	+.519	-.254	+.060	-.137	-.105
1.279	-.318	+.545	+.064	+.060	-.110	-.166
1.529	-.318	+.551	+.382	+.060	-.105	-.226

* Edge of bourrelet reaches membrane at $\varphi = -\frac{1}{2}\pi$
** Edge of bourrelet reaches membrane at $\varphi = +\frac{1}{2}\pi$

TABLE VI

The theoretical forces and torque on a 3" AP M79 projectile in a thin membrane.

Stage	θ_0	γ	η	Λ
(i)	$\geq 60°$	-.150	-.226	+.208
(ii)	60°	-.150	-.373	+.023
	65°	"	-.347	+.056
	70°	"	-.323	+.087
	75°	"	-.298	+.117
	80°	"	-.274	+.148
	85°	"	-.250	+.178
	90°	-.150	-.226	+.208
Stage	α	μ_1	μ_2	
(iii)	1.636	.706	.906	

TABLE VII

Theoretical plate penetration coefficients for the 3" AP M79 projectile in a thin membrane of STS at e/d = .03 and X' = 145000 (lb)/(in)^2. The effects of yaw and of rate of yaw.

$\theta_0 = \chi_0$	h_0'	$F(\frac{e}{d}, \theta)$	$\frac{1}{h_0'}\frac{\partial h_0'}{\partial \chi_0}$	$\frac{\partial h_0'}{\partial \chi_0'}$
0	.619	16000	.000	.00
15	.611	15800	-.019	+.33
30	.585	15100	+.066	+.64
45	.545	14100	+.298	+.91
...	...	...	...	...
60	.962	24900	+.124	+.51
65	.914	23700	+.058	+.38
70	.866	22400	+.014	+.28
75	.820	21300	-.009	+.20
80	.776	20100	-.018	+.13
84.8*	...	...	...	...
85	(.734)	(19000)	(-.013)	(+.06)
87.1**	...	...	...	...
90	(.692)	(17900)	(+.005)	(.00)

* Lower critical obliquity for cavitation

** Upper critical obliquity for complete penetration

Numbers in parentheses would be valid if there were no cavitation

TABLE VIII

Ballistic data for 3" Projectiles at 0° obliquity.

Projectile	Plate Number	Plate Tensile Strength	θ	χ	$\frac{e}{d}$	$F(\frac{e}{d},\theta)$
3" AP M79	X16835	132000	3°	0°	.431	42800±500
"	"	"	.5°	5°	.430	43200±200
"	"	"	3°	10°	.431	43600±200
3" Expr. (11 lb flat nose)	X16835	132000	0°	0°	.429	27600±200
"	"	"	.5°	5°	.431	26500±200
"	"	"	.5°	10°	.429	≥28400

APPENDIX B

LIST OF SYMBOLS

a	radius of the projectile.
A, C	moments of inertia of the projectile with respect to longitudinal and transverse axes.
b	length of nose from bourrelet to tip
α	dimensionless constant md^2/C.
β_1, β_2	dimensionless parameters of the undulation.
γ, η	dimensionless components of $f/(deX')$, with respect to axes which are parallel and normal to the plate.
d	diameter of the projectile.
e	thickness of the plate.
ε	infinitesimal quantity.
f	force on the projectile.
$F(e/d, \theta)$	plate penetration coefficient.
g, h	dimensionless coordinates of the center of mass.
g', g'', h', h''	derivatives with respect to q.
k_1, k_2, k_3, k_4	constants of integration.
i, j, k	orthogonal unit vectors with k perpendicular to the plate and with i perpendicular to the plane of incidence.
λ	distance from the center of mass to the edge of the bourrelet.
l	length of the projectile.
L	torque on the projectile.
Λ	dimensionless variable, $L/(d^2eX')$.
m	mass of the projectile.

LIST OF SYMBOLS (Continued)

μ_1, μ_2	constants of integration.
N	number of petals in a star crack.
$\mathbf{n}$	unit vector normal to the ogive.
ω	angular velocity of the projectile.
p	penetration of the projectile into the plate.
q	dimensionless variable $(\epsilon Y'/m)^{\frac{1}{2}}t$
r, φ	cylindrical polar coordinates of a point on the nose contour.
$\mathbf{r}$	position vector of a point on the nose contour.
R	radius of the ogive.
s'	effective area of an undulation.
t	time.
$\mathbf{t}$	unit vector tangent to the ogive.
θ	obliquity between the velocity of the projectile and the normal of the plate.
$\mathbf{v}$	velocity of the center of mass.
W	energy of the plate.
x, y, z	Cartesian coordinates of the center of mass of the projectile, with z the distance from the initial surface of the plate.
x, y	Cartesian coordinates of a point on the plate before plastic deformation.
x', y'	Cartesian coordinates of a point on the plate after plastic deformation.
Y'	effective yield stress.

LIST OF SYMBOLS (Continued)

$\dot{y}$, $\ddot{y}$, $\dot{z}$, $\ddot{z}$	derivatives with respect to time.
χ	angle between the axis of the projectile and the normal of the plate.
$\dot{\chi}$, $\ddot{\chi}$	derivatives with respect to t.
χ', χ''	derivatives with respect to q.
ξ	displacement of the center of the undulation.

APPENDIX C

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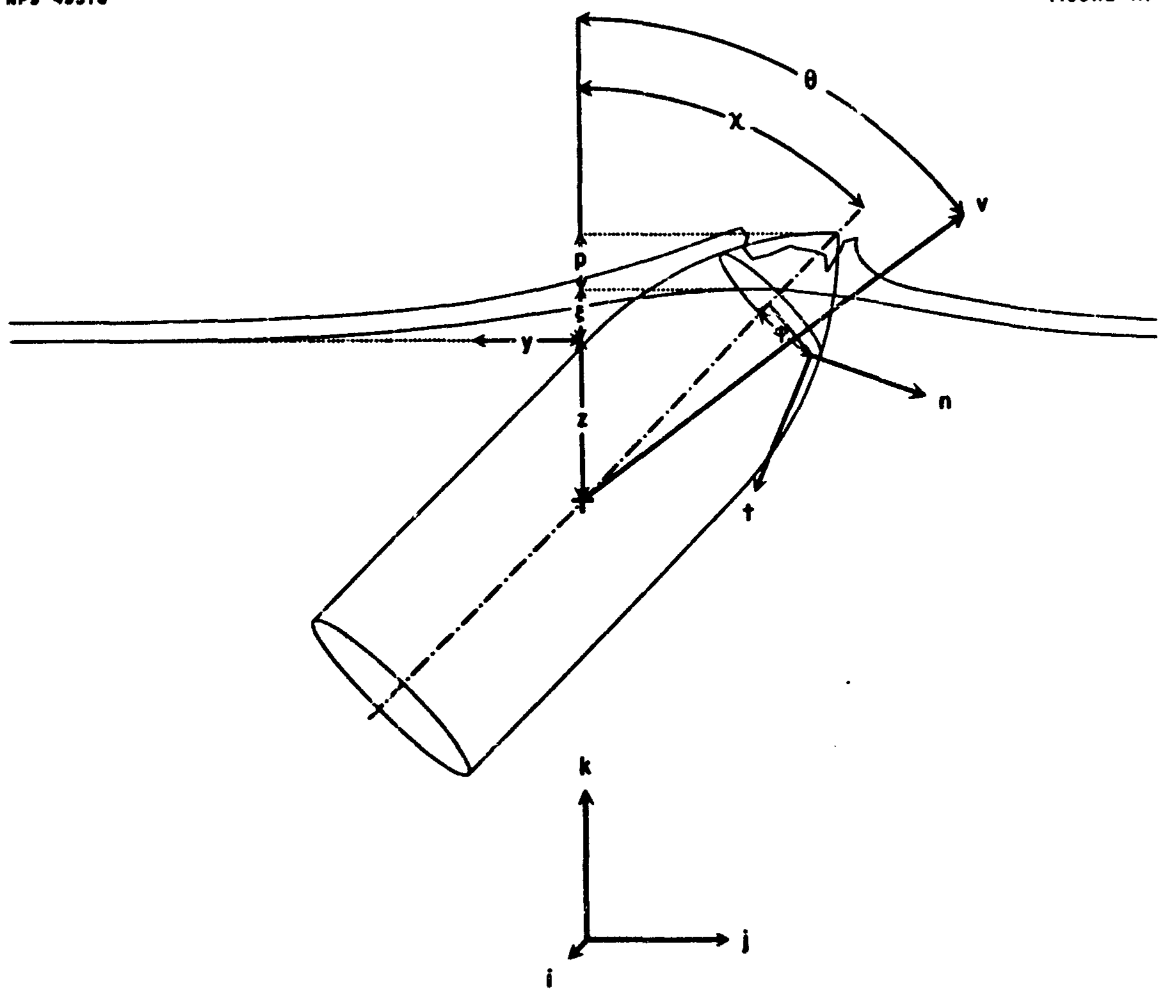
(2) Analytical Summary Part IV. The Theory of Armor Penetration. NPG Report No. 9-46.

(3) Analytical Summary Part V. Plastic Flow in Bars and Shells. NPG Report No. 954.

APPENDIX D

NPS 49376

FIGURE (1)

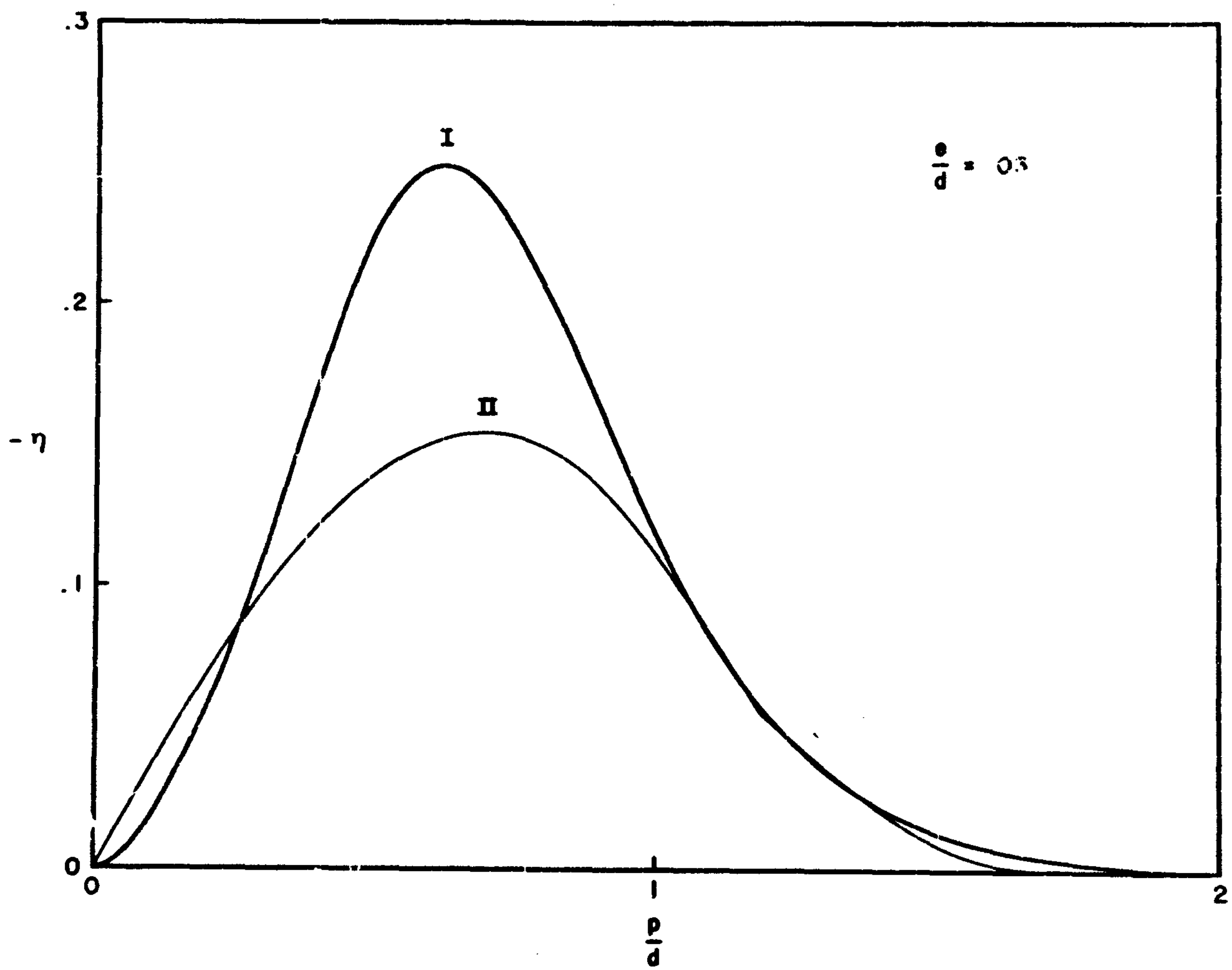


COORDINATES OF THE CENTER OF MASS AND ORIENTATION OF THE AXIS OF THE PROJECTILE

NPG 40379

FIGURE (?)

FORCE-PENETRATION CURVES FOR 3" AP M79 PROJECTILES AT 0°

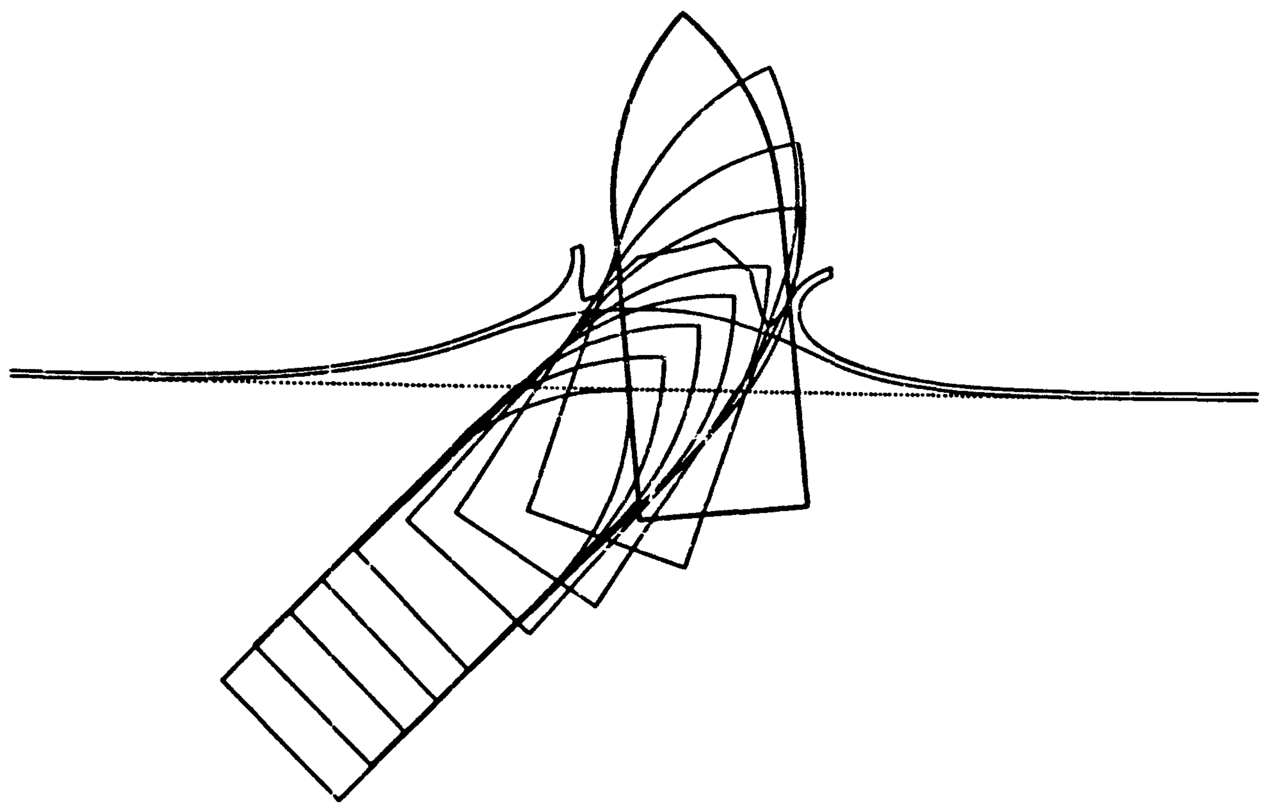


Curve I Experimental

Curve II Theoretical

NPG 49380

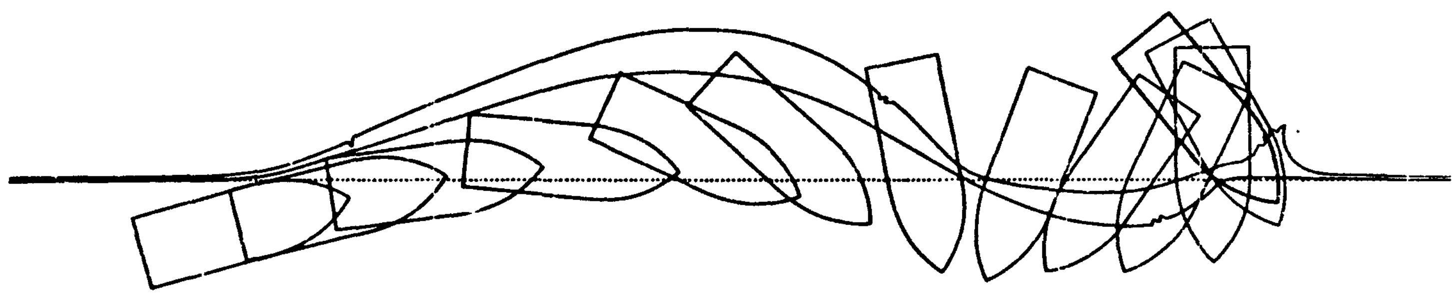
FIGURE (3)



THEORETICAL MOTION OF A 3" AP M79 PROJECTILE IN A LIMIT IMPACT AT 45° OBLIQUITY ON A MEMBRANE OF STS WITH A THICKNESS OF .03 CALIBERS

NPS 49381

FIGURE (4)

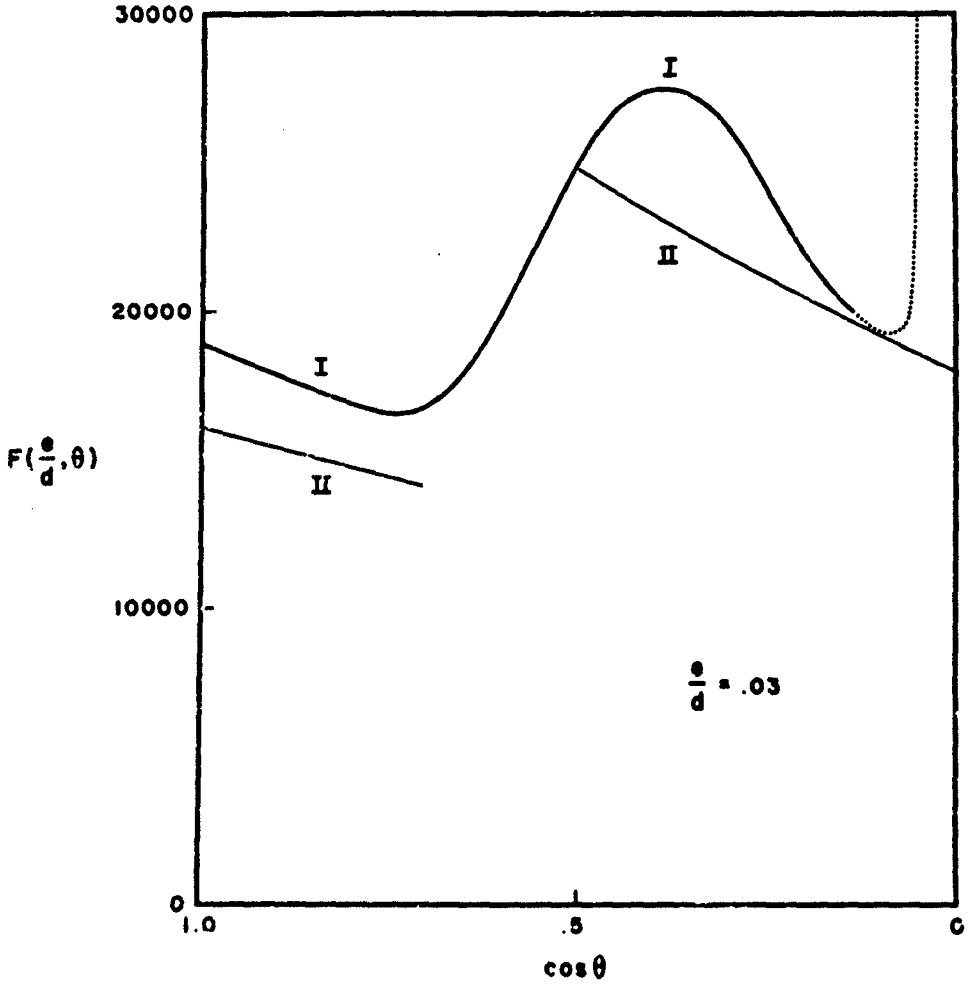


THEORETICAL MOTION OF A 3" AP M79 PROJECTILE IN A LIMIT IMPACT AT 75° OBLIQUITY ON A MEMBRANE OF STS WITH A THICKNESS OF .03 CALIBERS

NPG 49382

FIGURE ( )

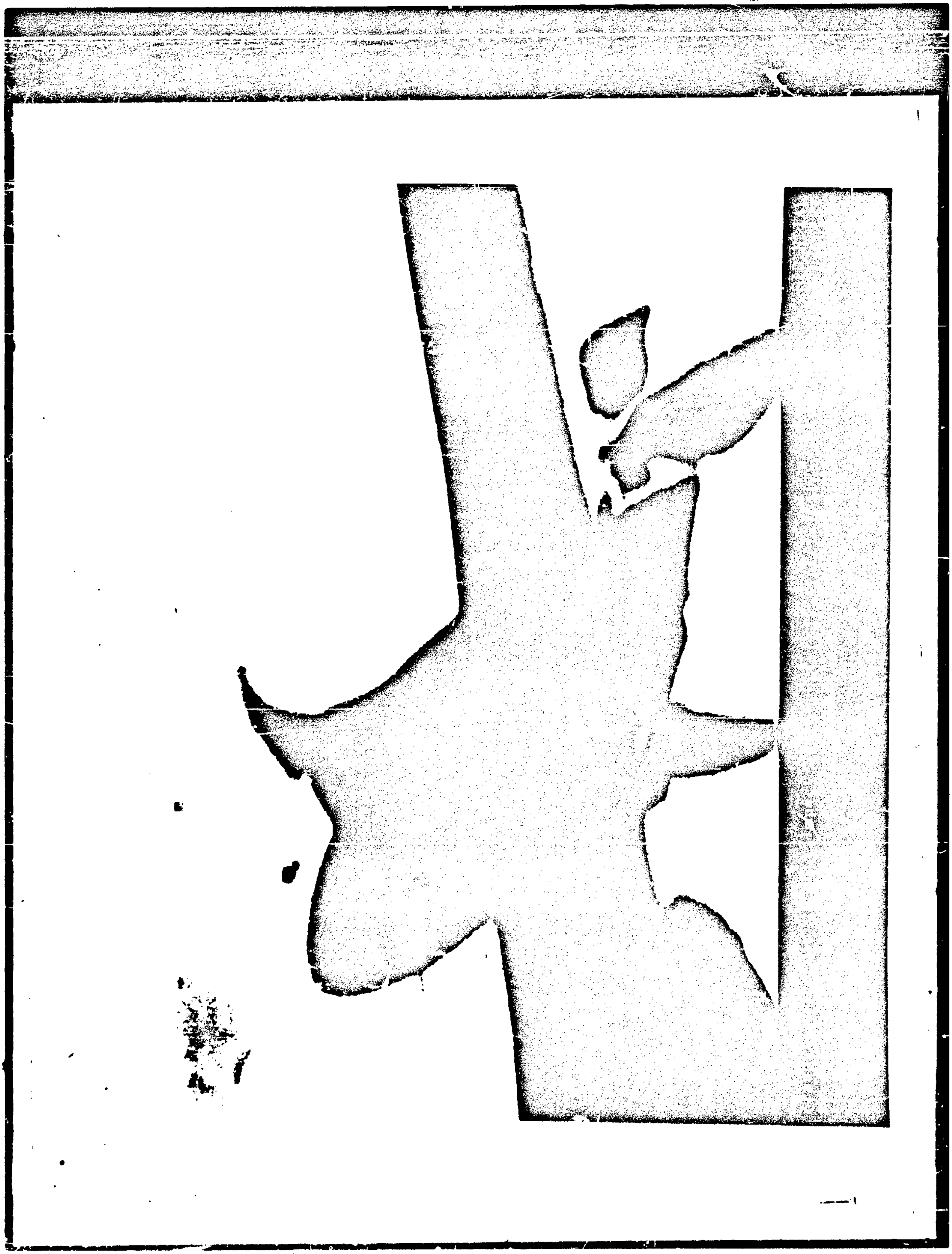
PLATE PENETRATION COEFFICIENTS FOR 3" AP M79 PROJECTILES IN STS

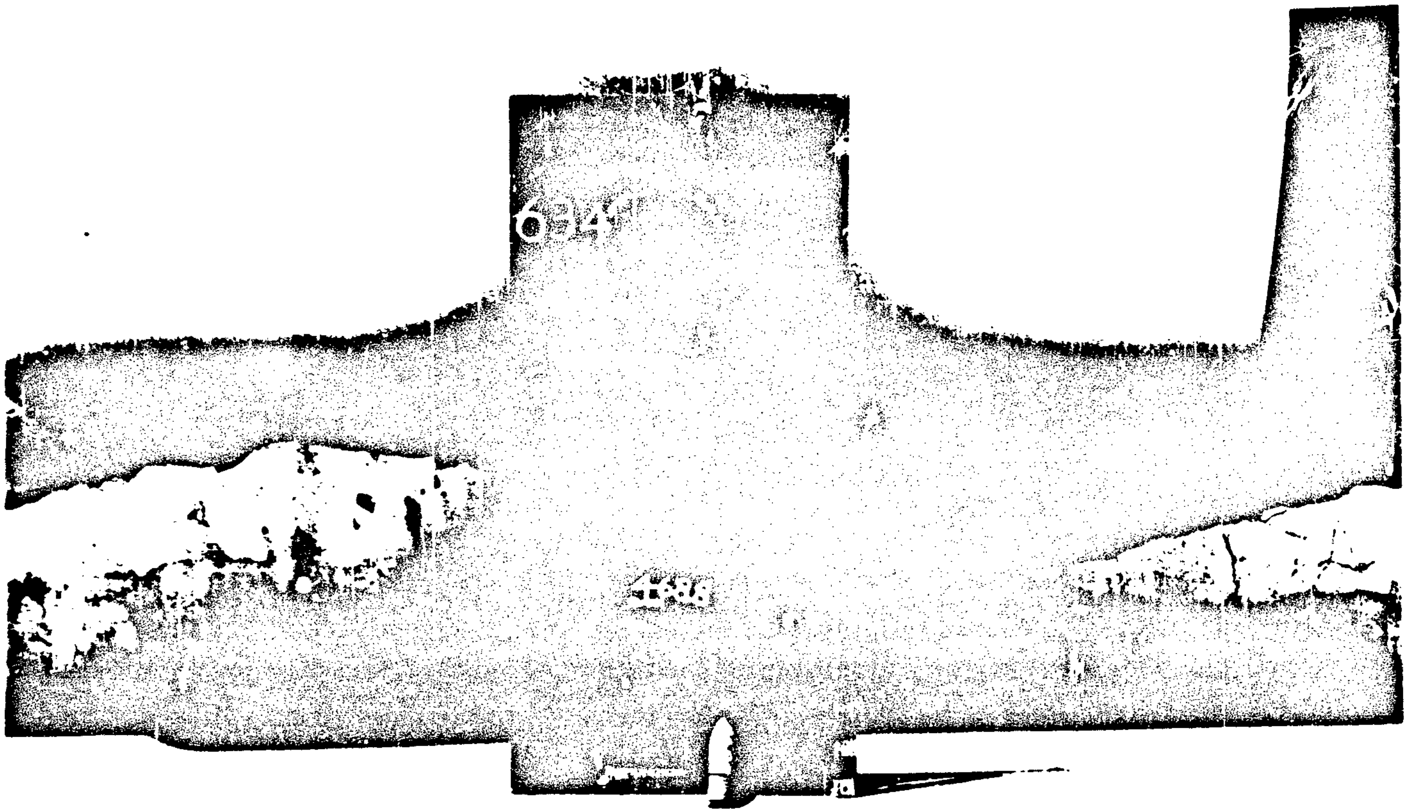


Curve I Standard ballistic function

Curve II Theoretical functions

FIG 5





NPG PHOTO NO. 3317 (APL). 27 May 1946 CONFIDENTIAL
APL Plate No. 614(0.25 STS C.I. No. 51724C) vs. 37MM M51B2 Y.S & Lock Co.
Uncapped Projectiles at 75° Obliquity. FRONT VIEW. See NPG Photo No. 3318 APL for Back View.

B.I.No.	"e"	"O"	S.V.f.s.	%	Pene.	Proj. Cond.
4688 APL	0.253	75°00'	2018	141	Inc.	Intact.

Fig 7

APPENDIX E

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THE THEORY OF PROJECTILE RICOCHET, by A. V. Hershey.
27 May 1955. 41p. append. A-E. (NPG Report No. 1041. Task assignment NPG E-11011-1). CONFIDENTIAL

Force-penetration curves for 3" AP M79 projectiles at low obliquity are derived with the aid of simplifying assumptions as to the stretching and folding of the petals of the plate. The theoretical curves are compared with experimental curves. An analysis is made of the effect of incomplete contact between the plate and the projectile. The analysis is guided by an experiment with an ogival indenter in aluminum foil.

The force and torque on a 3" AP M79 projectile at high obliquity are estimated with the aid of simplifying assumptions as to the plastic deformation at the sides of a crack.

1. Projectiles - Penetration
I. Hershey, A. V.
II. Title

NPG 104
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II. Title

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The force and torque on a 3" AP M79 projectile at high obliquity are estimated with the aid of simplifying assumptions as to the [illegible] [illegible] of the [illegible].

[illegible] curves are compared with experimental curves. [illegible] analysis [illegible] of the effect of incomplete contact between the plate and the projectile. The analysis is guided by an experiment wi[illegible] indenter in aluminum foil.

Th[illegible] force and torque on a 3" [illegible] projectile at high obliquity a[illegible] estimated w[illegible]h the aid of [illegible] assumptions as to the pl[illegible] deformation at the [illegible].